Metabolism: Glucokinase

- Physiological $\Delta G$ is $-27$ kJ mol$^{-1}$
How Does Equilibrium Change?

• Under physiological conditions, reaction is favorable – what about other conditions?

• We already know how to calculate changes in temperature, pressure:
  \[ dG = VdP - SdT \]

• What if we change the concentration of ATP?
Functions in Real Life

• **Think:** How do we use functions to characterize real-life experience?

• If we had a “Gibbs function,” we could calculate $G$ at one set of concentrations, and then at another: $\Delta G = G(N_{\text{ATP,2}}) - G(N_{\text{ATP,1}})$
First Step: Le Chatelier’s Principle

• Le Chatelier’s Principle (c. 1900):
  – Start at equilibrium
  – Change conditions (add ATP, change T, etc.)
  – Equilibrium will shift to counteract the change

• What does this mean?
  – Add ATP will favor the reaction which hydrolyzes ATP
  – Increase temperature will favor endothermic reaction (heat is a reaction product)
  – Increase pressure will favor product with smaller V
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Can we make this mathematically rigorous?
The “Gibbs” Function

• Consider a function that gives us the Gibbs energy:
  \[ G(P, T, N_{ATP}, N_G, N_{G6P}) \]

• Consider the differential:
  \[
  dG = V\,dP - S\,dT + \left( \frac{\partial G}{\partial N_{ATP}} \right)_{P,T,N_G,N_{G6P}} dN_{ATP} + \left( \frac{\partial G}{\partial N_G} \right)_{P,T,N_{ATP},N_{G6P}} dN_G
  \]

  \[
  + \left( \frac{\partial G}{\partial N_{G6P}} \right)_{P,T,N_{ATP},N_G} dN_{G6P}
  \]

• Or more simply:
  \[
  dG = V\,dP - S\,dT + \left( \frac{\partial G}{\partial N_{ATP}} \right) dN_{ATP} + \left( \frac{\partial G}{\partial N_G} \right) dN_G + \left( \frac{\partial G}{\partial N_{G6P}} \right) dN_{G6P}
  \]
Chemical Potential

• Define the chemical potential $\mu_x$:

$$
\mu_x = \left( \frac{\partial G}{\partial N_x} \right)_{\text{others const.}}
$$

• Then our differential becomes:

$$
dG = VdP - SdT + \mu_{\text{ATP}} dN_{\text{ATP}} + \mu_{\text{G}} dN_{\text{G}} + \mu_{\text{G6P}} dN_{\text{G6P}}
$$

• The red terms must tell us something about the non-PV work ($dw^*$)
Chemical Potential: Properties

• It’s a derivative: at a minimum, \( \frac{\partial G}{\partial N} = 0 \)

• They are inter-related: At equilibrium (dG = 0), a small increase in ATP (dN_{ATP}) must affect other dN’s

• It’s path-independent: because it’s derived from variables of state

• It’s intensive: Change in free energy per mole of X
Chemical Potential: Properties

• **Pure Substance**: At constant $T$, $P$, the $\mu$ for a pure substance is simple:

\[
dG = \mu_n dn_a
\]

\[
\frac{dG}{dn_a} = \mu_n
\]

— Therefore: $\mu$ is simply the partial molar Gibbs energy ($\Delta \tilde{G}$) for a pure substance
At Equilibrium: Open System

\[ aA + bB \rightarrow cC \]

- What is the Gibbs Energy exact differential?
“Open System” Implications

• **Implication #1:** Gibbs energy (at constant T, P) is simply the sum of chemical potentials of components

\[ G = \mu_a n_a + \mu_b n_b + \mu_c n_c + \cdots \]

• This is simply conservation of E (the first law) showing up again: **it applies to closed systems, too!**
“Open System” Implications

• **Implication #2:** If we’re interested in \( \Delta G \) (we are), adding a constant to \( \mu \) doesn’t matter:

\[
\begin{align*}
G_2 &= (\mu_{a,2})n_a \\
-G_1 &= (\mu_{a,1})n_a \\
\Delta G &= (\Delta \mu_a)n_a
\end{align*}
\]

\[
\begin{align*}
G_2 &= (\mu_{a,2} - \mu_a^0)n_a \\
-G_1 &= (\mu_{a,1} - \mu_a^0)n_a \\
\Delta G &= (\Delta \mu_a)n_a
\end{align*}
\]

• We can define a “standard state” chemical potential
Far From Equilibrium: Closed System

\[ aA + bB \rightarrow cC \]

- What is the Gibbs Energy exact differential?
  \[ dG = -SdT + VdP + \mu_a dn_a + \mu_b dn_b + \mu_c dn_c \]

- At constant T, P:
  \[ dG = \mu_a dn_a + \mu_b dn_b + \mu_c dn_c \]

- Up to this point, this is has simply been applied math. What does the chemistry say?
What is this $d\alpha$?

\[ \frac{dn_a}{a} = \frac{dn_b}{b} = -\frac{dn_c}{c} = -d\alpha \]

- A change per mole of reaction? What?!

- You’ve seen this before:

\[ 3O_2 + 2 \text{ Gly} \rightarrow 4\text{CO}_2 + 2\text{H}_2\text{O} + 2\text{NH}_3 \]

\[ \Delta H = -1160 \text{ kJ mol}^{-1} \]

— Enthalpy per mole of reaction
Approaching Equilibrium: Closed System

\[ aA + bB \rightarrow cC \]

• Express \( dG \) in terms of \( d\alpha \) (const. \( T, P \)):
  \[ dG = (c\mu_c - a\mu_a - b\mu_b)d\alpha \]

• Generalizing, we have another situation with “products minus reactants” (times moles)

• Closed system does not mean \( n \) cannot change! It does mean that \( n \) must change in a predictable way.
At Equilibrium: Closed System

\[ aA + bB \rightarrow cC \]

- At equilibrium, \( dG = d\alpha = 0 \), so therefore:
  \[ (c\mu_c - a\mu_a - b\mu_b) = 0 \]

- Additionally, the change in \( G \) per mole of reaction must be
  \[ \Delta \bar{G} = \frac{dG}{d\alpha} = (c\mu_c - a\mu_a - b\mu_b) \]

- Generalizing: products minus reactants (times moles)
What Does All This Mean?

• Used mathematical implications to define a new quantity, called the “chemical potential”

• For a pure substance chemical potential is simply molar Gibbs energy

• If we new chemical potentials of reactants and products, we could predict the change in Gibbs Energy (the “available” or “free” energy)