

SUMMARY

Covariance: $\sigma_{uv}^2 = \langle (u - \bar{u})(v - \bar{v}) \rangle$.

Propagation of errors: Assume $x = f(u, v)$:

$$\sigma_x^2 = \sigma_u^2 \left(\frac{\partial x}{\partial u} \right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v} \right)^2 + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u} \right) \left(\frac{\partial x}{\partial v} \right)$$

For u and v uncorrelated, $\sigma_{uv}^2 = 0$.

Specific formulas:

$$x = au \pm bv \quad \sigma_x^2 = a^2\sigma_u^2 + b^2\sigma_v^2 \pm 2ab\sigma_{uv}^2$$

$$x = \pm auv \quad \frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2} + 2\frac{\sigma_{uv}^2}{uv}$$

$$x = \pm \frac{au}{v} \quad \frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2} - 2\frac{\sigma_{uv}^2}{uv}$$

$$x = au^{\pm b} \quad \frac{\sigma_x}{x} = \pm b \frac{\sigma_u}{u}$$

$$x = ae^{\pm bu} \quad \frac{\sigma_x}{x} = \pm b\sigma_u$$

$$x = a^{\pm bu} \quad \frac{\sigma_x}{x} = \pm (b \ln a)\sigma_u$$

$$x = a \ln(\pm bu) \quad \sigma_x = a \frac{\sigma_u}{u}$$

Taken from *Data Reduction and Error Analysis in the Physical Sciences* by Bevington and Robinson, McGraw-Hill Publishers, 1992.